Grade 3 Mathematics, Quarter 3, Unit 3.1

Understanding Area as it Relates to Multiplication and Division

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Find the area of rectangles by multiplying whole number side lengths.
- Understand the product of the two sides is the area of the rectangle.
- Find the area of a rectilinear figure by decomposing it into two non-overlapping rectangles and adding the areas of the two smaller rectangles.
- Solve real world and mathematical problems involving rectangles with the same perimeter and different areas.
- Solve real world and mathematical problems involving rectangles with the same area and different perimeters.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Able to flow between contextual and non-contextual situations during problem solving and make meaning of numbers and symbols.
- Consider units involved.

Look for and make use of structure.

- Use a variety of strategies and properties to verify their answers.

Look for and express regularity in repeated reasoning.

- Does my answer make sense?

Essential questions

- How can you use multiplication to find the area of a rectangle?
- How is the area of a rectangle related to the product of the two sides?
- How can you decompose a plane figure into two rectangles to find the area of the plane figure?
- What are the different rectangles you can produce with a given perimeter in a real life situation?
- What are the different rectangles you can produce with a given area in a real life situation?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**3.MD**

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students in grade 2, partitioned a rectangle into rows and columns of same size squares and counted them. They also partitioned circles and rectangles into 2, 3, or 4 equal shares. They used addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns. They measured the length of an object by choosing and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. Students measured an object twice using different length units such as in, ft, cm, and m and to notice the difference in length between the units.

Current Learning

In grade 3, students find the area of a rectangle by multiplying the two side lengths and understand the product is the area of the rectangle. Students decompose a figure in to two rectangular shapes to determine the total area of the larger figure by adding the areas together. Students solve problems by showing rectangles with the same perimeter and different areas or the same area and different perimeters. This is at the developmental level.

Routines: Students will continue to practice multiplication and division facts to achieve fluency.

Future Learning

Students in grade 4 will know relative sizes of measurement units within one system of units including km, m, cm, ft, yd, and in. They will record measurement equivalents in a two column table. Students will apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Additional Findings

Principles and Standards for School Mathematics states, “the study of geometry in grades 3–5 requires thinking *and* doing. As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop. At the same time they are learning to reason, and to make, test, and justify conjectures about these relationships. This exploration requires access to a variety of tools, such as, graph paper, rulers, pattern blocks, geoboards, and geometric solids, and is greatly enhanced by electronic tools that support exploration, such as dynamic geometry software" (p. 165)

Progressions for the Common Core State Standards in Mathematics (draft) states that: "Initially students can use an intuitive notion of congruence (same size and same shape) to explain why the parts are equal…. Students come to understand a more precise meaning for "equal parts" as "parts with equal measurements." For example, when a ruler is partitioned into halve or quarters of an inch they see that each subdivision has the same length. In area models they reason about the area of the shaded region to decide what fraction of the whole represents.

The Common Core State Standards for Mathematics states that, "Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

Grade 3 Mathematics, Quarter 3, Unit 3.2

Understanding Fractions as Numbers

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Partition a whole into equal parts.
- Label the fractional part of a whole.
- Recognize that fractional parts make up the whole (i.e. 1/2 + 1/2 = 2/2 or 1).
- Decompose a whole into its fractional parts.
- Compose fractional parts into a whole.
- Represent fractions on a number line to show parts of a whole.

Mathematical practices to be integrated

Model with mathematics.

- Identify important quantities and their relationships and express these as a diagram or some kind of graphic organizer.

Reason abstractly and quantitatively.

- Attend to the meaning of quantities.

Attend to precision.

- Use clear definition and state the meaning of the symbols they choose consistently and appropriately.
- Strive for accuracy.

Essential questions

- What do the numerator and denominator in a fraction tell you?
- How can you accurately divide a whole into fractional parts?
- How can you accurately combine fractional parts to equal a whole?
- How can you use models, number lines or explanations to identify fractional parts of a whole?
- Where would you place a given fraction on the number line?
- Why did you place a given fraction in that location on the number line?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5] **3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students partitioned circles and rectangles into halves, fourths, and thirds. They were able to describe the shares as half of, or a third of halves or thirds. They were also able to describe the whole a 2/2, 3/3, and 4/4. They recognized that equal shares of identical wholes need not have the same shape.

Current Learning

Students in grade three partition a whole into equal parts. They understand a fraction a/b where a = the numerator which is the number of parts and b = the denominator which is the number of equal parts in the whole. They understand a fraction as a number on the number line and can represent fractions on a number line diagram. This is at the developmental level. The fractions sixths and eighths are added to their repertoire.

Routines: Students will continue to practice multiplication and division facts to achieve fluency.

Future Learning

In grade 4, students will identify equivalent fractions using visual fraction models, paying attention as to how the number and size of the parts differ even though the two fractions are the same size. The will also compare two fractions with different numerators and different denominators by creating common denominators or numerators. They will compare to benchmark fractions such a ½. They will use the symbols and words for greater than, equal to, and less than. They will also justify their conclusions using fractional models. They will add and subtract fractions with like denominators and solve word problems involving addition and subtraction of fractions.

Additional Findings

Progressions for the Common Core State Standards in Mathematics (draft) states that “Students build fractions from unit fractions, seeing the numerator 3 of ¾ as saying that ¾ is the quantity you get by putting 3 of the ¼ together. They read any fraction this way, and in particular there is no need to introduce “proper fractions” and “improper fractions” initially; 5/3 is the quantity you get by combining 5 parts together when the whole is divided into three equal parts.”

Principles and Standards for School Mathematics states, “Students should build their understanding of fractions as parts of the whole and as division. They will need to see and explore a variety of models of fractions, focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole and find equivalent fractions. They should

develop strategies for ordering and comparing fractions, often using benchmarks such as 1/2 and 1" (p.150).

Elementary and Middle School Mathematics: Teaching Developmentally states, "The first goal in the development of fractions should be to help children construct the idea of *fractional parts of the whole*—the parts that result when the whole or unit has been partitioned into *equal-sized portions or fair shares*" (p. 291).

The book also states that, when finding equivalent size of fractional pieces, " too often students see shapes that are already the same shape and size when they are asked questions about what fraction is shaded. The result is that the students think that equal shares might need to be the same shape, which is not the case. Young children, in particular, tend to focus on the shape, when the focus should be on equal *sized* pieces" (p. 293).

Additionally, the book says that "the focus on fractional parts is an important beginning, but number sense with fractions demands more—it requires that students have some intuitive feel for fractions. They should know 'about' how big a particular fraction is and be able to tell easily which of two fractions is larger. Like with whole numbers, students are less confident and less capable of estimating than they are at computing exact answers. Therefore, you need to provide many opportunities for students to estimate.

Even in daily classroom conversations" (p. 298). And "if children are taught these rules before they have had the opportunity to think about the relative sizes of various fractions, there is little chance that they will develop any familiarity with or number sense about fraction size" (p. 300).

Grade 3 Mathematics, Quarter 3, Unit 3.3

Comparing Fractions with Models and Reasoning

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand two fractions as equivalent if they are the same size or the same point on the number line.
- Recognize and generate simple equivalent fractions.
- Express whole numbers as fractions (i.e. 3 = 3/1).
- Recognize fractions that are equivalent to whole numbers (i.e. 4/4 = 1).
- Compare two fractions with the same numerator or the same denominator by reasoning about their size.
- Use ‹ , ›, or = to when comparing fractions.
- Recognize that comparisons are valid only when the two fractions refer to the same whole.

Mathematical practices to be integrated

Look for and make use of structure.

- Look for patterns to simplify.

Construct viable arguments and critique the reasoning of others.

- State your justification.
- Prove two ways.
- Ask "why" and seek an answer to that question.

Attend to precision.

- Use clear definition and state the meaning of the symbols they choose consistently and appropriately.
- Strive for accuracy.

Essential questions

- How can you prove that fractions are equivalent?
- What do you notice about equivalent fractions?
- How do you prove that a fraction is greater than, less than or equal to another fraction?
- If we make the denominator larger, what happens to the size of the parts into which the whole is divided?
- How can we express a whole number as a fraction?
- How can you identify fractions that are equivalent to a whole?
- Why is it important for the whole to be the same size when comparing fractions?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5]	**3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In grade 2, students partitioned circles and rectangles into halves, fourths, and thirds. They were able to describe the shares as half of, or a third of halves or thirds. They were also able to describe the whole a 2/2, 3/3, and 4/4. They recognized that equal shares of identical wholes need not have the same shape**.**

Current Learning

In grade 3, students understand that two fractions are equivalent if they are the same size, or the same point on the number line. They recognize and generate simple equivalent fractions.(e.g. ½ = 2/4, 4/6 = 2/3) They explain why the fractions are equivalent by using a visual fraction model. This is at the developmental level. They express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. This is at the reinforcement level. They compare two fractions with the same numerator or the same denominator by reasoning about their size. They use greater than, less than and equal to symbols to write comparisons. This is at the developmental level.

Routines: Students will continue to practice multiplication and division facts to achieve fluency.

Future Learning

In grade 4, students will identify equivalent fractions using visual fraction models, paying attention as to how the number and size of the parts differ even though the two fractions are the same size. The will also compare two fractions with different numerators and different denominators by creating common denominators or numerators. They will compare to benchmark fractions such a ½. They will use the symbols and words for greater than, equal to, and less than. They will also justify their conclusions using fractional models. They will add and subtract fractions with like denominators and solve word problems involving addition and subtraction of fractions.

Additional Findings

Principles and Standards for School Mathematics states, "Students should build their understanding of fractions as parts of the whole and as division. They will need to see and explore a variety of models of fractions, focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole and find equivalent fractions. They should develop strategies for ordering and comparing fractions, often using benchmarks such as 1/2 and 1" (p.150).

Elementary and Middle School Mathematics: Teaching Developmentally states that "the focus on fractional parts is an important beginning, but number sense with fractions demands more—it requires that students have some intuitive feel for fractions. They should know 'about' how big a particular fraction is and be able to tell easily which of two fractions is larger. Like with whole numbers, students are less confident and less capable of estimating than they are at computing exact answers. Therefore, you need to provide many opportunities for students to estimate. Even in daily classroom conversations" (p. 298). And "if children are taught these rules before they have had the opportunity to think about the relative sizes of various fractions, there is little chance that they will develop any familiarity with or number sense about fraction size" (p. 300).

Progressions for the Common Core State Standards in Mathematics (draft) states that "Although number line diagrams are important representations for students as they develop an understanding of a fraction as a number, in the early stages of the NF Progression they use other representations such as area models, tape diagrams, and strips of paper. These, like number line diagrams, can be subdivided, representing an important aspect of fractions. The number line reinforces the analogy between fractions and whole numbers."

PARCC Model Content Frameworks Mathematics Grades 3-11 states that, "Developing an understanding of fractions as numbers is essential for future work with the number system. It is critical that students at this grade level are able to place fractions on a number line diagram and understand them as a related component of their ever expanding number system."